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Examining spring wet slab and glide avalanche occurrence along the Going-to-the-Sun Road corridor, Glacier National Park, Montana, USA

Erich H. Peitzsch ^{a,*}, Jordy Hendrikx ^b, Daniel B. Fagre ^a, Blase Reardon ^a

^a U.S. Geological Survey, Northern Rocky Mountain Science Center, Global Change Building #4, Glacier National Park, West Glacier, MT, 59936, USA

^b Department of Earth Sciences, P.O. Box 173480, Montana State University, Bozeman, MT, 59717, USA

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ABSTRACT

Wet slab and glide snow avalanches are dangerous and yet can be particularly difficult to predict. Wet slab and glide avalanches are presumably triggered by free water moving through the snowpack and the subsequent interaction with layer or ground interfaces, and typically occur in the spring during warming and subsequent melt periods. In Glacier National Park (GNP), Montana, both types of avalanches can occur in the same year and affect the spring opening operations of the Going-to-the-Sun Road (GTSR).

We investigated the timing of wet slab and glide avalanche occurrence along the GTSR from 2003 to 2011 using meteorological and snowpack data from two high-elevation weather stations, one SNOTEL site, and an avalanche database to characterize 55 wet slab and 182 glide avalanches. Daily wet slab and glide avalanche occurrence were combined to represent an avalanche day and were compared to non-avalanche days (no avalanche occurrence) for 60 variables (both direct and derived measurements) using a univariate analysis. A classification tree (CART) was then trained to capture the most important variables for examining specific meteorological and snowpack variables that contribute to these types of wet snow avalanches. The CART was 10-fold cross validated using the data for 2003–2010 seasons and resulted in overall predictive accuracy of 73%. We then used the statistically optimal CART as a predictive model for the spring avalanche season of 2011, which resulted in an overall predictive accuracy of 82% for both avalanche and non-avalanche days, and a predictive accuracy of 91% for avalanche days.

The results suggest that the role of air temperature and snowpack settlement appear to be the most important variables in wet slab and glide avalanche occurrence. When applied to the 2011 season, the results of the CART model are encouraging and they enhance our understanding of some of the required meteorological and snowpack conditions for wet slab and glide avalanche occurrence.

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1. Introduction

In the USA most avalanche fatalities occur due to dry slab avalanches. However, wet snow avalanches, including both wet slab and glide avalanches, are also dangerous and can be particularly difficult to predict because they are relatively poorly understood (Baggi and Schweizer, 2009; Kattelmann, 1984; Reardon and Lundy, 2004; Reardon et al., 2006). Wet snow avalanches impact recreationists, transportation corridors, and ski areas. Because of their unpredictability, in some ski areas, poorly understood wet snow avalanches often create more difficulty for the ski area avalanche forecasters than better-understood dry snow avalanches (Savage, 2010). In addition, the mechanical properties of wet snow make both wet slab and glide

avalanches difficult to control with explosives (Clarke and McClung, 1999; Jones, 2004; Simenhois and Birkeland, 2010). While wet slab avalanches occur in all snow climate types, glide avalanches tend to be more common in maritime snow climates, though they can occur in drier snow climates during mid-winter thaws or in the spring (LaChapelle, 2001; McClung and Schaerer, 2006). With the anticipated increase of global mean temperatures due to climate change there are likely to be changes to the regional distribution of wet snow avalanches and a higher frequency of these types of avalanches. As temperatures rise rain-on-snow events might become more frequent, and the snowpack itself might trend toward a generally warmer and wetter one.

The focus of this study developed from a need for better understanding of wet snow phenomena during avalanche forecasting operations along the Going-to-the-Sun Road (GTSR), Glacier National Park, Montana, during spring road opening operations. Forecasting wet slab and glide avalanches is problematic given the current overly generalized meteorological and snowpack indicators. While wet loose avalanches affect operations along the GTSR, this type of wet snow

* Corresponding author at: USGS Global Change Building #4, Glacier National Park, West Glacier, MT, USA, 59936. Tel.: +1 406 888 7925; fax: +1 406 888 7923.

E-mail addresses: epeitzsch@usgs.gov (E.H. Peitzsch), jordy.hendrikx@montana.edu (J. Hendrikx), dan_fagre@usgs.gov (D.B. Fagre), blase.reardon@gmail.com (B. Reardon).

avalanche is less complicated, easier to predict, and generally less destructive. Thus, this paper concentrates on the conditions leading to wet slab and glide avalanches during late winter and spring (March 16–May 31).

The mechanisms driving wet slab avalanches contrast with those of dry slab avalanches. Wet slab avalanches depend upon the introduction of liquid water in the snowpack that changes the shear strength and decreases slope stability, whereas dry slab avalanches typically occur because of an increase in shear stress (Kattelmann, 1984). Conway and Raymond (1993) showed that the introduction of free water in the snowpack causes melting and disintegration of bonds between snow grains thus affecting slope stability. Bond disintegration occurs because of lateral spreading of water along a boundary such as a capillary barrier or ice layer. They also observed increased vertical strain during periods of water infiltration through a horizontal snowpack. Thus, it is possible that as grains metamorphose due to the presence of water on a slope this vertical strain leads to slope instability. Conway (1998) also suggests slab properties are affected by wetting of the snow surface and this contributes to slope instability and subsequent wet slab avalanches. Heywood (1988) suggested a different mechanism where water moves laterally along an impermeable boundary for a length of time and the liquid water content increases within the upper layer increasing creep and glide rates. He suggests this increase in velocity of the upper layer compared to the lower dry layer increases downslope shear stress and could result in a shear failure. The instability is released through an avalanche, refreezing of ponded water at the impermeable boundary, drainage out the bottom of the snowpack, or melting of the overlying slab itself thereby reducing stress (Kattelmann, 1984).

Glide is the process during which the snow cover on a slope slips downhill along the interface with the underlying ground (Jones, 2004). When glide rates vary on a slope, a tensile fracture, commonly called a glide crack, forms upslope of the area of faster glide where stresses are concentrated (Clarke and McClung, 1999; Jones, 2004; LaChapelle, 2001). Full-depth avalanches often follow the formation of a glide crack (Fig. 1). Such glide avalanches are difficult to predict, however, because not all glide cracks culminate in avalanches (Reardon et al., 2006), and for those that do, the time between crack formation and avalanche release can vary widely, ranging from several hours to weeks or even months (McClung and Schaerer, 2006).

Jones (2004) reviewed glide processes and glide avalanches, relying heavily on a model formulated by Clarke and McClung (1999). As presently understood, glide has three prerequisites: (1) a snowpack-ground interface with little roughness, such as bare rock or grass, (2) a temperature of 0 °C at the interface, which allows liquid water to

exist, and (3), a slope angle greater than 15°. This combination reduces the effects of friction and increases the influence of liquid water at the interface. The amount and distribution of liquid water present at the interface between the snowpack and the ground are thus understood to be the critical influences on glide rates and glide avalanches. Free water within the snowpack may also contribute to glide avalanches by decreasing the viscosity of the slab, allowing it to flow over surface irregularities. The primary sources for this water are rain-on-snow events and snowmelt, which can occur at the surface due to short wave radiation or air temperature, or at the interface as a result of stored heat in the ground. Glide avalanches are more often a concern for operational avalanche forecasting programs, particularly highway and railroad programs, because they can occur repeatedly in the same paths, often annually and sometimes within the same season (Clarke and McClung, 1999; Reardon and Lundy, 2004; Simenhois and Birkeland, 2010; Stimberis and Rubin, 2004; Wilson et al., 1996).

Forecasting wet slab and glide avalanches relies on local experience and monitoring of local meteorological conditions (Jones, 2004). Baggi and Schweizer (2009) completed an analysis of wet snow avalanches in a small valley in the Swiss Alps. They focused their study on all wet snow avalanches throughout the winter and spring with a sub-focus on wet slab avalanches. While the physical failure processes differ between wet slab and glide avalanches, both types of avalanches are dependent upon free water flowing either through the snowpack or at the ground-snow interface, which is driven by meteorological parameters. Snowpack structure and water flow through the snowpack are presumably important for wet slab and glide avalanche occurrence, yet obtaining such vital data is often limited due to access to starting zones along the GTSR because of terrain. Therefore, in addition to using limited snowpack data, monitoring meteorological metrics (i.e. air temperature, net radiation, rain) would be more useful if relationships between meteorological parameters and wet slab and glide avalanche occurrence were quantified. Thus, establishing a link between wet slab and glide avalanche release and meteorological data will aid in wet avalanche forecasting and improve safety on the GTSR.

The purpose of this study was to improve forecasting of natural wet slab and glide avalanches along the Going-to-the-Sun Road corridor. The primary objective of this study was to examine measurable relationships, if any, between various meteorological and snowpack metrics and wet slab and glide avalanche occurrence by comparing differences between avalanche days and non-avalanche days, and to apply these results directly to an operational forecasting program. A secondary objective was to determine whether there exists a difference in these meteorological and snowpack parameters between wet slab and glide avalanches. Because similar processes drive both types of wet snow avalanches, determining whether a relationship exists or not would aid forecasters in distinguishing between days when a wet slab or glide avalanche, or both, may occur.

2. Study area and methods

2.1. Location

Wet slab and glide avalanches occur regularly in the mountains of Glacier National Park (GNP), U.S.A. Some of these pose a threat during the annual spring opening of the GTSR (Reardon and Lundy, 2004). This two-lane, 80-kilometer road traverses the park, crossing the Continental Divide at Logan Pass (2026 m a.s.l.). The Park closes a 56 km section of the road each winter due to inclement weather, heavy snowfall, and avalanche hazards. Since 2003, GNP and the U.S. Geological Survey (USGS) have partnered to provide an operational forecasting program for the annual spring opening of the GTSR. Forecasters from the program maintain two automated weather stations and record weather data and snow and avalanche



Fig. 1. Full depth glide avalanche on Heavens Peak, Glacier National Park, MT 15 June 2010.

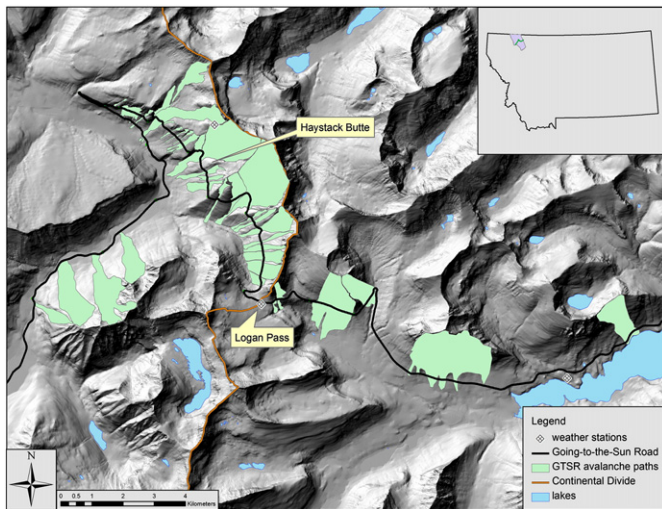


Fig. 2. Overview of study area – avalanche paths along the GTSR corridor, Glacier National Park, MT, USA.

observations in a database developed specifically for the site. The avalanches recorded in this database comprise a rare, multi-year dataset of natural wet snow avalanches from a well-instrumented drainage.

The study area was comprised of the slopes visible from the GTSR west of the Continental Divide (Fig. 2). These slopes were located in the headwaters of McDonald Creek upstream of Avalanche Creek, an area of 24,761 ha. The lowest point in the study area lies at 1036 m and the surrounding peaks reach 2915 m. McDonald Valley is the major drainage in the park west of the Continental Divide, and Logan Pass is both the high point of the GTSR and the lowest point of the Continental Divide in the drainage.

2.2. Climate and weather

The snow avalanche climate of the study area exhibits a generally maritime precipitation regime accompanied by continental temperature characteristics (Mock and Birkeland, 2000; Reardon et al., 2006). The contrasting precipitation and temperature regimes are due to the study area's position astride the Continental Divide, which allows both Pacific and continental air masses to influence the area's weather. Average annual precipitation is 2083 mm at Flattop SNOTEL (1810 m a.s.l.). Snowfall and rain amounts during the spring tend to be light, unless winter weather patterns persist into spring, leading to dramatically wet and stormy weather. Though the peak snow water equivalent (SWE) typically occurs the last week in April (1970–2010) the snowpack is at its most variable during the spring (Klasner and Fagre, 2000). Peak SWE totals can vary by more than 50% and the date of peak SWE has occurred as early as March 27 and as late as May 27. Summer-like periods of fair weather typically occur each spring when a ridge of high pressure (or anticyclone) directs warm air into the region on a southwest flow. Under a strong ridge, these conditions can persist for a week or more. Two notable features of this pattern are nighttime inversions and daytime temperatures that peak higher on successive days. Both features result in temperatures that are warmer in upper elevation start zones than at midslope or valley bottoms.

The site's mid-hemisphere latitude (48° 40' N) plays an important role in spring-time weather and avalanches. It makes for longer days of ever-more intense insolation as the snow removal season progresses. The length of daylight doubles from late December to late June (8.2 to 16.2 h); average monthly radiation increases 7.5 times in a similar period (1.05×10^8 J/m² in December to 7.95×10^8 J/m² in July) (Finklin, 1986).

3. Data sources

3.1. Avalanche data

The source for wet and glide avalanche observations and measurements was the database created for the GTSR avalanche forecasting program (Reardon and Lundy, 2004). From the database, we selected all natural avalanches identified as wet slab and glide avalanches. The database yielded 55 distinct wet slab and 182 distinct glide avalanches between March 16 and June 13 from eight seasons (2003–2011). These 182 glide avalanche events occurred on 85 distinct days and the 55 wet slab avalanche events occurred on 19 distinct days. We refer to each individual day in which any wet slab or glide avalanche occurs as an avalanche day. In total, 237 distinct avalanche events on 104 avalanche days were observed. On five avalanche days both wet slab and glide avalanches were observed. Records included date of occurrence and destructive class for all 237 avalanches, as well as starting zone elevation for 233 of the 237 avalanches. Vertical fall was available for 212 of the 237 avalanches, and start zone aspect was recorded for 234 of the 237 avalanches. Of 86 avalanches that occurred in avalanche paths affecting road operations 29 of those 86 reached the road or beyond. The database did not include data describing slope angle of the avalanche starting zones nor an explicit identification of whether snowmelt or rain triggered the avalanches. A large subset of this database of events (seasons 2003–2010) with 93 total avalanche days was used for the univariate analysis and to train the CART. A smaller subset of the database (only the 2011 season) with 11 avalanche days was used to test the CART.

The database consisted of field observations and measurements of snow conditions, avalanche occurrence, and weather conditions collected and recorded using standard methods and nomenclature (Greene et al., 2004). Though direct measurements were taken when possible, most topographic parameters were estimated in the field and later verified using photographs and topographic maps. All data were collected and recorded by a total of five observers over nine years. Field observations were collected during operational hours, typically weekdays from 0700 to 1600. Snow removal and forecasting operations began the first week of April and continued generally through the early part of June.

Several factors complicated field observations. One was the fact that observations were collected opportunistically, as part of an operational forecasting program, rather than systematically as might be expected in a pre-designed study. Thus, there are few observations from weekends. Avalanches that occurred on weekends were assigned to one of the two weekend days according to apparent age of the avalanche debris and crowns when observed on subsequent workdays. A second was the large size of the study area. Many avalanche sites were inaccessible or not visible until snow removal permitted travel to slopes above the upper reaches of the GTSR. Pre-season overflights of the study area in 2004 and 2006 provided a baseline for observations. In all nine seasons, most slopes within the study area were visible by the second week of April. The exception was the southwest facing slopes between Haystack Butte and Logan Pass, which were visible by late April or early May each season (Fig. 2). Despite the constraints, observations occurred most days and included most of the study area on any given day. Finally, a change in observers may have also added error due to individual subjectivity in identifying and classifying avalanches, but there was overlap in observers throughout the nine seasons. We conclude the database contained the majority of the wet and glide avalanches that occurred in the study area each late winter and spring.

3.2. Meteorological data

Meteorological data were collected at two automated weather stations (AWS) and one United States Department of Agriculture National Resource Conservation Service (NRCS) Snowpack Telemetry (SNOTEL)

Table 1
60 variables derived from nearby AWS (Logan Pass and Garden Wall) and SNOTEL site (Flattop). Due to missing data in some of the parameters, this resulted in an uneven number of avalanche and non-avalanche days per parameter. However, in most instances, the numbers of cases for each parameter are similar.

Metric	n		Metric	n	
	av	nonav		av	nonav
Logan Pass AWS (elev. 2035 m)			Flattop SNOTEL site (elev. 1810 m)		
Min. air temperature (°C)	83	82	Snow water equivalent change 1 day (cm)	88	88
Min. air temperature 1 day (°C)	83	82	Snow water equivalent change 2 day (cm)	88	86
Min. air temperature, avg. 3 day (°C)	85	82	Snow water equivalent change 3 day (cm)	88	86
Min. air temperature, avg. 5 day (°C)	85	78	Snow water equivalent change 4 day (cm)	88	85
Min. air temperature, difference 1 day (°C)	81	79	Snow water equivalent change 5 day (cm)	88	81
Min. air temperature, difference 2 day (°C)	81	79	Snow water equivalent change 6 day (cm)	88	80
Min. air temperature, difference 3 day (°C)	81	78	Total snow depth (cm)	88	88
Sum positive min. air temperature 1 day (°C)	54	26	Snow depth change 1 day (cm)	88	88
Sum positive min. air temperature 3 day (°C)	70	37	Snow depth change 2 day (cm)	88	86
Sum positive min. air temperature 5 day (°C)	72	44	Snow depth change 3 day (cm)	88	86
Max. air temperature (°C)	83	82	Snow depth change 4 day (cm)	88	85
Max. air temperature 1 day (°C)	83	82	Snow depth change 5 day (cm)	88	81
Max. air temperature, avg. 3 day (°C)	85	82	Snow depth change 6 day (cm)	88	80
Max. air temperature, avg. 5 day (°C)	85	78	Snow density (%)	88	88
Max. air temperature, difference 1 day (°C)	81	79	Rain proxy (mm)	83	83
Max. air temperature, difference 2 day (°C)	81	79	Days since isothermal proxy	83	88
Max. air temperature, difference 3 day (°C)	81	78	Garden Wall AWS (elev. 2240 m)		
Sum positive max. air temperature 1 day (°C)	80	62	Avg. net radiation (W/m ²)	36	43
Sum positive max. air temperature 3 day (°C)	85	69	Avg. net radiation 1 day (W/m ²)	36	43
Sum positive max. air temperature 5 day (°C)	85	66	Avg. net radiation 3 day (W/m ²)	36	43
Mean air temperature (°C)	83	82	Avg. net radiation 5 day (W/m ²)	36	41
Mean air temperature 1 day (°C)	83	82	Max. net radiation (W/m ²)	36	43
Mean air temperature, avg. 3 day (°C)	85	82	Max. net radiation 1 day (W/m ²)	36	43
Mean air temperature, avg. 5 day (°C)	85	78	Max. net radiation 3 day (W/m ²)	36	43
Mean air temperature, difference 1 day (°C)	81	79	Max. net radiation 5 day (W/m ²)	36	41
Mean air temperature, difference 2 day (°C)	81	79	Min. net radiation (W/m ²)	36	43
Mean air temperature, difference 3 day (°C)	81	78	Min. net radiation 1 day (W/m ²)	36	43
Sum positive mean air temperature 1 day (°C)	68	46	Min. net radiation 3 day (W/m ²)	36	43
Sum positive mean air temperature 3 day (°C)	79	54	Min. net radiation 5 day (W/m ²)	36	41
Sum positive mean air temperature 5 day (°C)	82	59			
Wind speed avg. (m/s)	84	85			
Relative humidity (%)	83	82			

site. The Garden Wall Weather Station (GWWX) sits atop a southwest-facing slope at 2240 m just west of the Garden Wall, a rock spine that forms the Continental Divide. The station was situated within 10 km of most of the wet slab and glide avalanches included in this study. The Kipp and Zonen CNR1 Net Radiometer is located at this station. The station was installed in December, 2003 and operated during the 2004–2007 and 2009–2011 seasons. A second AWS was located at Logan Pass Visitor Center at an elevation of 2035 m and operated during all nine seasons with a suite of sensors similar to GWWX. Logan Pass is broad, low-angle, gentle terrain at tree line. At both stations, temperature measurements were made at sixty second intervals and reported as hourly values. These hourly data were then used to calculate daily average, minimum, and maximum values. Occasional missing data occurred at both stations due to instrument and power problems. Because the period of record at Logan Pass AWS was more consistent than GWWX AWS, all temperature metrics are from Logan Pass. The use of Logan for forecasting operations and a strong correlation coefficient ($r=0.99, p<0.01$) between average daily temperature at both sites justifies the use of temperature measurements from this station only.

Precipitation was measured at Flattop Mountain SNOTEL (1810 m) in the headwaters of the McDonald Valley. The station is located below the summit of a broad plateau between the two major mountain ranges in GNP. It is operated by the NRCS and is part of the SNOTEL network. It provided daily SWE and height-of-snow (HS) measurements, and rain measurements (United States Department of Agriculture, 2011).

3.3. Data analysis

The quality of the meteorological and avalanche data was verified through visual methods as well as filtering out outliers based on

parameters from nearby meteorological stations. A total of 60 meteorological variables were derived from the AWS, and a univariate analysis was completed examining avalanche days (both wet slab and glide avalanches combined) and non-avalanche days (Table 1). Mean, maximum, and minimum daily data were calculated from the hourly data from 0000 to 2359 inclusive. A univariate analysis (Wilcoxon rank sum test) was completed for each variable comparing avalanche and non-avalanche days (Wilcoxon, 1945). An avalanche day was classified as such if there was at least one wet-slab or glide avalanche that occurred within that 24 hour period. No additional weighting or scaling was applied to days with more avalanche activity. The non-avalanche days were randomly selected from days between March 16 and May 31 (the period of recorded avalanche activity), and were equally distributed by month, to match the monthly distribution of avalanche days, over each season from 2003 to 2010. Where data permitted, this resulted in an equal number of avalanche and non-avalanche days for every month and every season, and for the data set as a whole. Due to missing data in some of the parameters, this resulted in an uneven number of avalanche and non-avalanche days per parameter; however, in most instances, the numbers of cases for each parameter are close. A level of significance $p=0.05$ was utilized to determine if differences between avalanche and non-avalanche days were statistically significant.

The significantly different variables resulting from the univariate analysis of avalanche and non-avalanche days were then selected as input variables in a classification tree (CART) analysis. A full discussion of CART can be found in Breiman et al. (1993). The data used in the CART analysis were slightly modified from those used in the univariate analysis. CART analysis does a case-wise deletion of missing data, so if one parameter has a blank or erroneous data value, then the entire case (in this case avalanche/non-avalanche day) is removed

Table 2

From a total of 60 direct and derived variables, 46 variables were found to be significantly different between avalanche days (av) and non-avalanche days (nonav). Due to missing data in some of the parameters, this resulted in an uneven number of avalanche and non-avalanche days per parameter. However, in most instances, the numbers of cases for each parameter are similar.

Metric	Median		p-value	Metric	Median		p-value
	av	nonav			av	nonav	
Min. air temperature (°C)	1.2	-1.9	<0.001	Mean air temperature, avg. 5 day (°C)	2.2	0.5	0.002
Min. air temperature 1 day (°C)	0.6	-2.1	<0.001	Mean air temperature, difference 1 day (°C)	1.2	0.1	0.004
Min. air temperature, avg. 3 day (°C)	0.0	-2.2	<0.001	Mean air temperature, difference 2 day (°C)	2.4	0.0	0.03
Min. air temperature, avg. 5 day (°C)	-0.7	-2.1	0.01	Mean air temperature, difference 3 day (°C)	2.6	-0.2	0.009
Min. air temperature, difference 1 day (°C)	1.0	-0.4	0.02	Sum positive mean air temperature 1 day (°C)	9.1	3.6	<0.001
Min. air temperature, difference 2 day (°C)	1.4	-0.3	0.046	Sum positive mean air temperature 3 day (°C)	12.2	7.2	<0.001
Min. air temperature, difference 3 day (°C)	2.6	-0.7	0.02	Sum positive mean air temperature 5 day (°C)	15.5	10.4	0.004
Sum positive min. air temperature 1 day (°C)	5.4	2.6	0.005	Snow water equivalent change 1 day (cm)	-0.5	-0.2	0.02
Sum positive min. air temperature 3 day (°C)	6.3	3.3	0.01	Snow water equivalent change 2 day (cm)	-1.5	-0.8	0.02
Sum positive min. air temperature 5 day (°C)	7.7	5.0	0.02	Snow water equivalent change 3 day (cm)	-2.2	-1.4	0.01
Max. air temperature (°C)	7.3	2.0	<0.001	Snow water equivalent change 4 day (cm)	-2.8	-1.3	0.008
Max. air temperature 1 day (°C)	7.0	2.9	<0.001	Snow water equivalent change 5 day (cm)	-3.3	-1.3	0.004
Max. air temperature, avg. 3 day (°C)	6.0	3.4	<0.001	Snow water equivalent change 6 day (cm)	-3.7	-1.5	0.002
Max. air temperature, avg. 5 day (°C)	5.2	3.5	0.001	Snow depth change 1 day (cm)	-4.3	-2.5	0.003
Max. air temperature, difference 1 day (°C)	1.6	-0.4	0.006	Snow depth change 2 day (cm)	-7.8	-5.2	0.008
Max. air temperature, difference 2 day (°C)	2.3	-0.1	0.009	Snow depth change 3 day (cm)	-10.6	-6.8	0.002
Max. air temperature, difference 3 day (°C)	3.4	-0.2	0.003	Snow depth change 4 day (cm)	-13.4	-8.9	0.002
Sum positive max. air temperature 1 day (°C)	14.1	7.5	<0.001	Snow depth change 5 day (cm)	-17.8	-9.7	<0.001
Sum positive max. air temperature 3 day (°C)	23.9	13.9	<0.001	Snow depth change 6 day (cm)	-18.1	-11.4	0.002
Sum positive max. air temperature 5 day (°C)	31.5	20.4	<0.001	Max. net radiation (W/m ²)	279.3	158.5	0.009
Mean air temperature (°C)	4.2	-0.2	<0.001	Max. net radiation 1 day (W/m ²)	233.6	171.3	0.04
Mean air temperature 1 day (°C)	3.7	0.2	<0.001	Min. net radiation 5 day (W/m ²)	-79.4	-69.4	0.01
Mean air temperature, avg. 3 day (°C)	2.9	0.5	<0.001	Relative humidity (%)	64.5	87.0	<0.001

from the analysis. Given the missing data present in our meteorological and snowpack data, we filled these blanks with the mean value of all the available data for that parameter. This approach was employed to ensure that all days could be considered and not excluded from the CART analysis. After De'Ath and Fabricius (2000), the risk of creating potential bias by including mean values for missing data was assessed to be less than the value of the potentially excluded data. CART has been applied to avalanche forecasting by a number of others, for both predictive (Eckert and Latif, 1997; Hendrikx et al., 2005; Jones and Jamieson, 2001) and exploratory data analysis (Baggi and Schweizer, 2009; Davis and Elder, 1994; Davis et al., 1996, 1999; Schweizer and Jamieson, 2003). CART has a number of distinct advantages over other statistical methods for discriminating or grouping of data. CART methods are non-parametric and are largely insensitive to underlying distributions and the results are often more easily understood than other statistical output (Davis et al., 1999).

A classification tree was permitted to grow using the avalanche and non-avalanche data for the period from 2003 to 2010 and the 46 significant parameters as defined from the univariate analysis (Table 2). The tree was grown, recursively splitting data into increasingly homogenous nodes using the Gini index (Breiman et al., 1993). Similar to the approach by Hendrikx et al. (2005), we initially permitted the tree to continue to grow so that all nodes were homogenous with only avalanche day or non-avalanche day present (i.e. an over-fitted tree). Hendrikx et al. (2005) notes that these types of trees have been used as an exploratory tool (Davis et al., 1996, 1999; Rosenthal et al., 2002), but due to the over-fitting, they are of limited use for predictive, or forecasting, purposes.

Therefore, once this tree had been grown to its maximum extent, a 10-fold cross validation technique was employed to reduce the tree to a more statistically defensible level for predictive purposes. The technique described by Breiman et al. (1993) and employed by Hendrikx et al. (2005) and Baggi and Schweizer (2009) essentially uses randomly drawn samples from their data to repeatedly grow the tree. The end result is a tree that uses only the splits that provide the maximum average correct classification (or minimum misclassification cost) across all 10 trees. For this analysis we did not adjust the misclassification costs, and for either case (i.e. false positive and false negative) they were kept equal at one. The statistically best tree

was selected from the set of cross validated trees, which minimized the combined cost of the cross validation and resubstitution costs. Breiman et al. (1993) provide an in-depth discussion on this selection process, while Hendrikx et al. (2005) present a short summary with respect to an avalanche data set. We then used this tree for the prediction of avalanche days from our initial data set (2003–2010) and for testing against our 2011 avalanche season. To achieve this, a random number of non-avalanche days equal to the number of avalanche days (in each month) were selected and the tree was tested against those non-avalanche days as well as the overall 2011 season from May 15 to June 30. The period of record was extended through June during the 2011 season to capture a prolonged spring and late-season avalanche occurrence.

4. Results

4.1. Univariate results

The average day of wet slab avalanche occurrence from 2003 to 2010 seasons was April 21 and the average day of glide avalanche occurrence was May 3; a non-significant difference. The univariate analysis of avalanche days (both wet slab and glide avalanches combined) versus non-avalanche days produced 46 variables of significant difference (Table 2). All of the air temperature and snow water equivalent change variables were significantly different between avalanche and

Table 3

From a total of 60 direct and derived variables, 8 variables were found to be significantly different between wet slab (ws) and glide avalanche days (gs).

Metric	n		Median		p-value
	ws	gs	ws	gs	
Snow water equivalent change 5 day (cm)	15	78	-1	-3.95	0.049
Snow water equivalent change 6 day (cm)	15	78	-1	-4.45	0.03
Total snow depth (cm)	15	78	271.3	234.1	0.04
Snow depth change 6 day (cm)	15	78	-8.1	-20.2	0.01
Snow density (%)	15	78	38	42.0	0.006
Avg. net radiation 3 day (W/m ²)	8	30	8.85	24	0.04
Avg. net radiation 5 day (W/m ²)	8	30	5.05	24.75	0.03
Days since isothermal proxy	12	75	20.0	44.0	0.01

Table 4

Classification matrix for 10 fold cross validated tree, with observed cases compared with predicted. Shaded values indicate correct prediction.

N = 176		Observed	
Avalanche day = 88		Avalanche day	Non-avalanche day
Non-avalanche day = 88			
Predicted	Avalanche day Non-avalanche day	61 20	27 68

non-avalanche days. On average, all air temperature values were higher for avalanche days than for non-avalanche days. The change in snow water equivalent (SWE) was, on average, greater from one to six days prior to avalanche occurrence for avalanche days than non-avalanche days. Total snow depth was not significantly different, but the daily and cumulative daily (two to six days prior) change in snow depth differed significantly between avalanche and non-avalanche days. Maximum net radiation, maximum net radiation averaged over the day before with the day of avalanche occurrence, and minimum net radiation averaged over the preceding five days and the day of avalanche occurrence were the only radiation variables that differed significantly. Relative humidity was also found to differ between avalanche and non-avalanche days. The univariate analysis of wet slab versus glide avalanche days produced eight variables of significant difference (Table 3). The sample size of wet slab avalanches was notably less than those of glide avalanches. Six of those eight variables were snowpack variables and the remaining two were radiation variables.

4.2. CART results

For the CART analysis we used 10 fold cross validation to determine the final tree for prediction, resulting in a tree with three splits and four terminal nodes, and had an overall accuracy of 73% (129 of 176 cases correctly identified). The probability of detection (POD) for avalanche days was slightly lower at 69% with 61 of the 88 cases correctly identified (Table 4).

The tree split the 176 cases of avalanche (1) and non-avalanche days (0) (Node ID 1) first on the maximum daily air temperature at Logan Pass (`airtemp_C.logan.max`) where a threshold of $+5.35$ °C discriminated between mainly avalanche days (1) (Node ID 3) above this threshold and non-avalanche days (0) (Node ID 2) below this threshold. The node (ID 2) of mainly non avalanche days (0) was then further split by the mean daily air temperature at Logan Pass (`airtemp_C.logan.mean`) where a threshold of $+1.65$ °C discriminated between mainly avalanche days (1) (Node ID 5) above this threshold and non-avalanche days (0) (Node ID 4) below this threshold. The node (ID 5) of mainly avalanche days was then further split by the change in snow depth over five days at Flattop SNOTEL site (`snowdepth_change5day`) where a threshold of -19.95 cm discriminated between mainly avalanche days (1) (Node ID 20) below this threshold – i.e. more snow depth loss- and non-avalanche days (0) (Node ID 21) above this threshold (Fig. 3).

The above cross validated classification tree was then used to hind-cast avalanche days in the 2011 spring season. The 2011 season was not used in the training or cross validation of the final tree, so this analysis provides a sense of the “true” predictive power of this tree, rather than the statistically estimated predictive power. Of the 11 cases of avalanche days in the 2011 season, our tree accurately predicted 10 of them. Overall, our tree predicted 23 avalanche days from March 16 to June 30 when applied to the entire spring 2011 season. It misclassified 13 actual non-avalanche days as avalanche days and one avalanche day as a non-avalanche day. However, when we repeat the analysis undertaken with the training data and select 11 random non-avalanche days from the 2011 spring season (with the same monthly distribution as the avalanche days), the tree only misclassified 3 of the days as avalanche days. Thus, for this season, this tree has an overall accuracy of 82% and an accuracy of predicting avalanche days, or POD, of 91%.

5. Discussion

This study focused on wet slab and glide avalanche occurrence and selected associated meteorological and snowpack conditions during

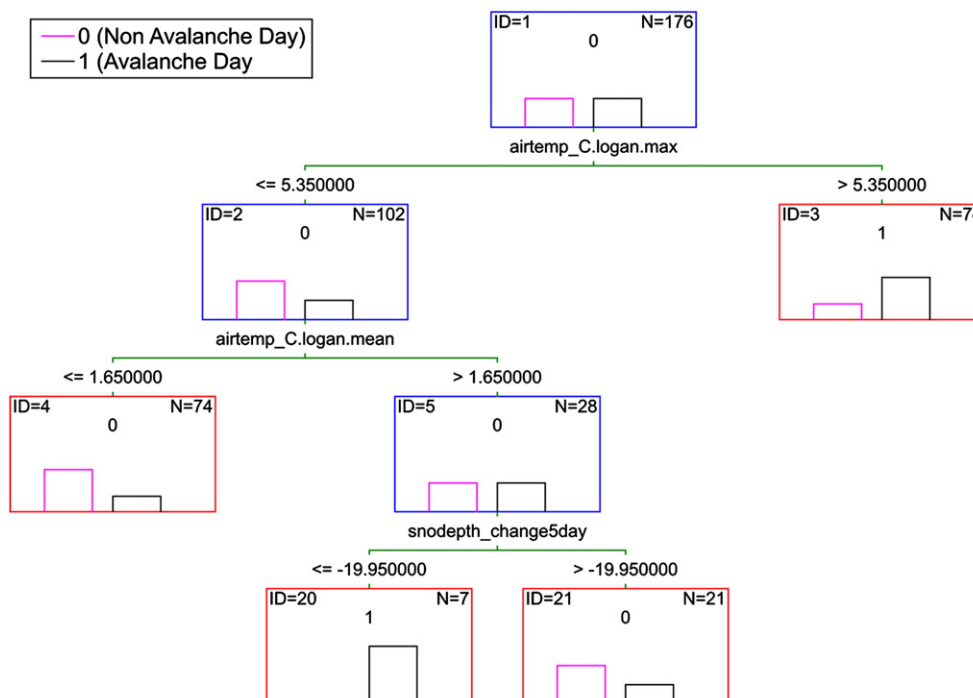


Fig. 3. Avalanche day classification tree following 10 fold cross validation, where 0 = non-avalanche day and 1 = avalanche day. Node ID is shown in the top left of each node, while number of cases in each node is shown at the top right.

late-winter and spring in Glacier National Park, Montana, USA. The purpose of this study was to improve forecasting of natural wet slab and glide avalanches along the Going-to-the-Sun Road corridor, and was completed under the premise that the results would be directly applied to this particular operational setting. Therefore, the direct and derived variables used in the analysis were variables that avalanche practitioners would find easily accessible during avalanche forecasting operations.

5.1. Temperature and snowpack variables

The 46 variables determined to be significantly different between avalanche and non-avalanche days in the univariate analysis included all of the temperature metrics studied and a few of the snowpack variables (Table 2). *Baggi and Schweizer (2009)* found that minimum air temperature and the sum of positive air temperatures over 3 or 5 days are the temperature variables best associated with wet slab avalanche occurrence. The temperature metric results in our study are consistent with this finding. However, many more of our temperature metrics were found to be significantly different. This may be attributed to this dataset consisting of spring avalanche occurrences only, while *Baggi and Schweizer (2009)* included more of the winter season.

Larger decreases in snow water equivalent for avalanche days suggest that more water is moving through the snowpack on days when wet slab and glide avalanches occur. While Flattop SNOTEL site is approximately 350–500 m below most of the avalanche starting zones, a time lag was not apparent between loss of water in the snowpack at Flattop and avalanche occurrence at higher elevations. This may be due to the fact that the starting zones along the GTSR corridor are predominantly southwest facing and may be similar to the timing of water flow through the snowpack at a flat, sub-alpine site like Flattop SNOTEL. Thus, using Flattop SNOTEL site as one tool for assessing water flow through the snowpack in the starting zones may be appropriate and useful.

The greater change, on average, in snow depth from one to six days prior to and including avalanche days than non-avalanche days is also similar to findings by *Baggi and Schweizer (2009)*. They found that a change in snow depth over three days is greater during avalanche days. Snow depth change over five days was not only significant in our univariate results, but it was also present as the third and final node in our CART. While monitoring snow depth changes during forecasting operations was done, it was typically used for monitoring settlement rates and overall snowpack consolidation. Thus, perhaps a slightly greater focus on snow depth change at Flattop SNOTEL site would be beneficial in helping to forecast wet slab and glide avalanches along the GTSR corridor.

5.2. Net radiation and relative humidity

While three radiation variables (maximum net radiation, maximum net radiation averaged over one day prior to and including avalanche day, and minimum net radiation averaged over 5 days prior to and including avalanche day) were significantly higher on avalanche days than non-avalanche days, the sample sizes were substantially smaller than other variables thus requiring caution when interpreting the net radiation results. Therefore, while it appears that wet slab and glide avalanche days have higher radiation values than non-avalanche days (except in the case of minimum net radiation over 5 days) and will continue to be monitored during forecasting operations, further work is needed to accurately assess the association of net radiation values and avalanche occurrence.

Finally, relative humidity was less on avalanche days than on non-avalanche days. This suggests that wet slab and glide avalanche occurrence is more likely with dry conditions. The dataset used in the univariate analysis did not include rain-on-snow events as all wet

slab and glide avalanches occurred during warm days with ample solar input.

5.3. Wet slab and glide comparison

Splitting the dataset and comparing wet slab to glide avalanche days decreased the sample sizes dramatically. Thus, while significant differences in eight variables between wet slab and glide avalanches exist, it is important to note the small sample size used in this comparison. Since 46 variables were significant in avalanche and non-avalanche days and only eight variables differed between wet slab and glide avalanche days, it appears that these two types of wet snow avalanche occur under similar conditions with subtle differences. Based on these results and field observations, it appears that glide avalanches tend to occur when the snowpack is more mature and drainage channels are established. This is reinforced by the presumed mechanics of these types of avalanches (*McClung and Schaerer, 2006*). While wet slab avalanches can occur at any interface in the snowpack, glide avalanches occur at the ground-snow interface. This requires free water moving along that interface and this typically happens when the snowpack is more mature and later in the spring. Further work on glide avalanches specifically is needed to better predict their occurrence.

5.4. Use of operational classification tree

The application of a CART to this dataset resulted in an overall predictive accuracy of 73% with three splits. These are similar results to the CART generated by *Baggi and Schweizer (2009)* that resulted in a 71% accuracy for wet snow avalanche activity. The splits in their tree were on: index of capillary barriers, the days since the isothermal state was reached, and the 3-day sum of positive air temperatures. Our splits occurred on maximum daily temperature, average daily temperature and the change in snow depth over five days before and including the avalanche day. The overall accuracy percentage of the CART in this study was lower than work by *Hendriks et al. (2005)*, but their work was a more direct action avalanche environment and also included dry slab avalanches.

The first two splits in the tree indicate that both maximum and mean temperature influence wet slab and glide avalanche occurrence. Given the widely held assumption that sustained non-freezing temperatures contribute to wet snow instability, it is surprising that minimum temperature was not directly evident in the tree. However, the mean daily temperature threshold on the split occurs above the freezing level and this appears to capture, albeit indirectly, the notion that periods of sustained, above freezing temperatures may contribute to wet slab and glide avalanche release.

The final split of the tree illustrates that snowpack properties are important variables associated with wet slab and glide avalanches. A loss of snow depth greater than 19.95 cm over five days prior to the avalanche reflects settlement and melt processes as the snowpack transitions to a more consolidated spring snowpack. Since Flattop SNOTEL site sits at an elevation below the majority of starting zones along the GTSR it can be used as a forecasting tool, but similar values of snow depth loss in the starting zones may be quite different.

The use of mean values for missing data for the CART analysis was a pragmatic decision, as the use of 46 variables, each with different missing data, would have led to an extensive loss of usable data for the initial training. The influence of these data points on the final CART is trivial as there were only 13 days (~7%) where at least one of the three variables used in the final CART had missing data and had to be replaced with a synthetic data point. Furthermore, these synthetic data points assumed the average value for all data points (both avalanche and non-avalanche day) for each parameter so as to reduce potential bias (*De'ath and Fabricius, 2000*).

The 2011 season was abnormal in that the total SWE at Flattop was 117% of average (42 year period of record) at the beginning of our analysis dates on March 15 and 340% of average for June 30. It was a fairly active spring avalanche season with both wet slab and glide avalanches occurring, but both occurring notably later than normal. The onset of the first wet slab avalanche cycle was May 10, and the first recorded glide avalanche occurred on May 15. Despite late avalanche occurrences because of a cool, wet spring there were warming periods similar to other years. Thus, it was deemed appropriate to use the CART generated from the historical dataset (2003–2010) to hindcast the 2011 season. Jones and Jamieson (2001) utilized data excluded from the training of their classification tree to assess the predictive ability of their model for skier-triggered dry slab avalanches. Building upon that knowledge we utilize a CART model that has been cross-validated with additional new data from natural, wet snow avalanches to assess its “true” predictive power.

For the 2011 season the CART performed better than the statistical estimates of predictive power would have suggested. The 10 fold cross validated tree estimated an overall accuracy of 73% with a POD for avalanche days of 69%. However, the CART achieved considerably better results in 2011 for both overall accuracy and avalanche day POD with 82% and 91% respectively. Jones and Jamieson's (2001) application of their model predicted approximately two-thirds of the actual skier-triggered, dry slab avalanche days. These results are encouraging as they show the high degree of predictive power using this statistical technique on a new data set from the same location. We are unable to compare these results directly to those of others, as these are the first results that have used a trained CART and then assessed its predictive power on a new (not just cross validated) data set of natural, wet snow avalanches.

Operationally, these results translate to one avalanche day that would have gone unpredicted, and 13 days that road clearing operations would have been altered because of predicted avalanche occurrence. While these results are encouraging, it must be recognized that the sample size for the 2011 season is quite small and that this year had above average snow depths. However, the spring avalanche season is a relatively short period of time, thus prohibiting a large sample size within a year, and on a yearly basis the model would only be used for this short period of time.

6. Conclusion

This study examines contributory meteorological and snowpack variables of wet slab and glide avalanche occurrence along the GTSR. The data set used for this study is unique in that it documents natural wet slab and glide avalanches over nine spring seasons in an area with an established highway avalanche forecasting program. Statistical analyses included a univariate analysis of 60 directly measured and derived meteorological and snowpack variables. The variables found to be significant in this analysis were then used in a 10 fold cross validated classification tree. This tree was then used on a new data set, the 2011 season, to hindcast avalanche days with remarkable success resulting in an overall accuracy of 82% with a POD for an avalanche day at 91%. These results are very encouraging and while they cannot be directly compared to previous work, and we urge caution due to the small sample size, we remain encouraged about the high POD for avalanche days. We have found these methods effective and encourage future researchers to consider a similar approach when using CART to enable a more “true”, opposed to statistical, estimate of the predictive ability of their selected tree. The results generated in this study will be directly applied in avalanche forecasting operations as general guidance. However, caution must be used, as many avalanche practitioners are well aware, Perla's adage states that “the only rule of thumb in avalanche work is that there is no rule of thumb” (McClung and Schaerer, 2006).

The results from this study suggest that the role of air temperature, snowpack settlement, and SWE loss appear to be the most important variables in wet slab and glide avalanche occurrence. The CART shows that maximum and mean temperatures, as well as a change in snow depth, are variables that should be monitored during avalanche forecasting operations. However, forecasting wet slab and glide avalanches is a difficult problem. More process based studies, such as the utilization of lysimeters to understand water flow processes, in-situ monitoring of glide rates and snowpack settlement, and the examination of snowpack structure properties, are necessary in the future and would likely improve our understanding of these types of wet snow avalanches.

Disclaimer

Any use of trade, product, or firm names is for descriptive purposes only and does not imply endorsement by the U.S. Government.

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