SUPPLEMENTARY MATERIALS

Estimation of Annual Predation Rate

We estimated the number of moose killed annually by presuming that: (*i*) average daily kill rate observed in January and February was representative of the average kill rate from December through April, and (*ii*) kill rates from May through September were 8% of that observed during the upcoming winter, and (*iii*) kill rates during October and November were 21% of that observing during the upcoming winter.

Together, these assumptions correspond to an average daily kill rate, averaged throughout the year, which is 50% of the average daily kill rate observed during January and

10   February. That is, the daily predation rate, averaged throughout the year, is reasonably estimated as 50% of the daily predation observed during mid-winter. This value is also comparable to previously proposed adjustments, based on different lines of reasoning (Messier, 1994).

The appropriateness of assumption (*i*) does not require kill rates do not be constant throughout the period between December and April for the assumption to be reasonable. Indeed, kill rates in March are likely greater than in December (Smith *et al.* 2004). Assumption (*i*) is reasonable so long as the average kill rate from December through March is reasonably characterized by that observed in January and February. Assumptions (*ii*) and (*iii*) are justified for two reasons. First, of the 1,294 male moose that were at least one year of age that we

20   autopsied between 1959 and 2008, 6.3% died during the period of antler growth (early May to mid-September), and 16.5% died during the period when moose have fully grown antlers (mid-September to late November). These proportions correspond to the seasonal adjustments described above (e.g., 0.21 = 0.165/0.772, where 0.772 is the proportion of male moose dying

without antlers of any kind, i.e., $1 - 0.165 - 0.063$).   Second, although wolves certainly kill

many calves (<6 months old) between late-May and late-November, our estimates of annual

predation, moose abundance, and survival rate are intended to include only these moose, which

are >6 months of age. We considered moose <6 months of age not to have been recruited into

the population. Ultimately, to estimate annual predation rate, we divided the estimated number

of recruited moose killed during the year by the number of moose in the population.

30   Recent work from YNP also provides a basis for estimating annual predation rate from

kill rate that is observed during the late winter. In YNP per capita kill rate on recruited elk (>6

months old) has been estimated for several packs over several years, during the months of

November, December, March, May, June, and July (see Metz et al. 2011 [PLoS ONE 6(3):

e17332]). Kill rate was estimated by monitoring wolves wearing VHF or GPS collars.These

estimates suggest kill rates during May were, on average, 94% of those observed during winter

(November through March); and kill rates during June and July were 44% of those observed

during Winter. If summer kill rates are similar from June through September, and and kill rates

in October are intermediate between summer and early winter kill rates (November); then an

average daily kill rate, averaged throughout the year, would be 78% of the average daily kill

40   rate observed during winter. This value is similar toMessier's (1994) conversion of 71%.

Theoretical Analyses

Each section below represents predator-prey dynamics governed by Equation (2) and various

specific forms of numerical response (NR) and functional response (FR) for dynamics.

          Linear NR; Type II, prey-dependent FR. – Suppose dynamics are governed by a linear

numerical response, $P = \alpha_0 + \alpha_1 N$, and a functional response equal to $KR = \beta_0 N/(\beta_1 + N)$. To

understand how KR and PR are related, begin, as in any case, with what is true by definition: $PR=KR \times N/P$. Into this equation replace P with $\alpha_o + \alpha_1 N$ and KR with $\beta_o N/(\beta_1+N)$. Algebraic manipulation shows thatPR $= \beta_0(\alpha_o + \alpha_1 N)/(\beta_1+N)$. Because the denominators of KR and PR

50 are identical, the relationship between KR and PR can be understood by examining the numerators. Because the numerators of KR and PR are both linear functions of N, it is the case that KR and PR are linearly related.

Asymptotic NR; Type I, prey-dependent FR. – Now suppose the numerical response is asymptotic, $P = \alpha_o N/(\alpha_1+N)$, and the functional response is linear and prey-dependent, KR=aN. In this case, PR=$a\alpha_o N/(\alpha_1+N)$. Because PR increases asymptotically with N and KR increases linearly with N (i.e., KR=aN), it is the case that PR increases asymptotically with increases in KR (Fig. 1).

Asymptotic NR; Type I, prey-dependent FR. – If dynamics are governed by an asymptotic numerical response (as in the previous case) and an asymptotic, prey-dependent

60 functional response, then KR increases asymptotically with N (i.e., KR $= \beta_o N/[\beta_1+N]$), and the relationship between PR and N involves a quadratic function, i.e., PR $= \alpha_o \beta_o N/[\alpha_o \beta_o + (\alpha_o+\beta_o)N + N^2]$. These relationships with N, mean that KR and PR have a unimodal relationship (Fig. 1).

Type I, ratio-dependent FR. –When predator-prey dynamics involve kill rates that increase linearly with the ratio of prey to predator (i.e., Type I, ratio-dependent functional response, where KR $= aN/P$), then PR is independent of KR. To see how, recall that PR=KR $\times P/N$, which reduces to PR=a, when KR is replaced with aN/P. This result holds for both linear and asymptotic numerical responses.

Type II, ratio-dependent FR. –When predator-prey dynamics involve an asymptotic, ratio-dependent functional response, KR= $aN/(P+bN)$, the relationship between KR and PR is

70   negative. Specifically, PR is equal to $a - bKR$. To see how this result arises, begin with the

identity:

$$aP\frac{N}{N} = aP$$

Then divide both side of this equation by $1/(P+bN)$, which gives:

$$\frac{aN}{(P+bN)}\frac{P}{N} = \frac{aP}{(P+bN)}.$$

Notice that the left side of the equation is equal to PR:

$$PR = \frac{aP}{(P+bN)}.$$

Add and subtract abN to the numerator of the right side of equation:

$$PR = \frac{aP + abN - abN}{(P+bN)}.$$

Then express the right side of the equation in terms of $aN/(P+bN)$:

80      $$PR = a - b\left(\frac{aN}{P+bN}\right).$$

Notice that $aN/(P+bN)$ is equal to KR:

$$PR = a - bKR.$$

Because b and KR are always positive values, KR and PR are negatively related. This result

arises for both linear and asymptotic numerical responses.

Simulation Analyses

To generate realistic simulation results, we set parameters to values based on the IRNP system

(i.e., r_{int}=0.20, a_1=0.41, and a_2=0.62; Peterson *et al.* 1998, Vucetich & Peterson 2004). The

prey-dependent functional response that we used was KR = $6N/[500+N]$, and the ratio-

90   dependent functional response was KR $= 0.234N/(P+2.71\times10^{-2}N)$ (Vucetich *et al.* 2002). We

used these parameter values and with $s_1= s_2 =s_2 = 0$, we set K to the value which yielded mean

population abundances similar to that observed in IRNP (i.e., 24 wolves and 1000 moose).

Specifically, we set K=2500 for simulations where θ=1, K=2000 for simulations with prey-

dependency and θ=4, and K=1300 for simulations with ratio dependency and θ=4.

     We ran sets of simulations with $s_1 = 1.2$ and $s_3 = 0.12$. These values approximate

observed values (Vucetich *et al.* 2002; Vucetich & Peterson 2004). With these values fixed,

s_2=0.06 yielded simulated trajectories with coefficients of variation comparable to those

observed in the real populations (i.e., CV_{wolf}=0.37, CV_{moose}=0.39). To consider sets of

simulations where the functional response represents a much smaller source of stochasticity, we

100   set s_1=0.3.

     To avoid inadvertent influence of initial dynamics, each correlation was calculated

after each simulation had run for 400 years.

Table S1 Model selection results corresponding to Figure 3. SSR is the residual sum of squares, and dAICc is the AICc for the model of interest minus the smallest AICc for the set of models being considered, where AICc is Akaike's information criterion, corrected for small sample size. By definition, the best model has a dAICc of zero. R^2 is equal to one minus the SSR divided by the total sum of squares. Each panel of figure three depicts the best-fitting models from this table, if the best fitting model also had a p-value<0.05.

Site	Model	Kill-rate/predation-rate relationship				Predator:prey-ratio/predation-rate relationship			
		SSR	ΔAICc	R^2	p-value	SSR	ΔAICc	R^2	p-value
ISLE ROYALE	Linear	0.10216	99.7	0.02	0.34	3.83×10^{-2}	0	0.63	<0.001
	Logarithmic	0.10413	100.5	0.00	0.68	4.59×10^{-2}	6.9	0.56	<0.001
	Quadratic	0.00845	0	0.08	0.20	3.83×10^{-2}	124.0	0.63	<0.001
	Power	0.11761	105.5	0.01	0.57	3.95×10^{-2}	1.2	0.62	<0.001
	Exponential	0.0865	92.9	0.03	0.27	3.93×10^{-2}	1.0	0.63	<0.001
YELLOWSTONE	Linear	4.16×10^{-3}	1.7	0.19	0.18	3.66×10^{-4}	1.1	0.88	<0.001
	Logarithmic	4.28×10^{-3}	1.4	0.17	0.21	7.96×10^{-4}	9.7	0.74	<0.001
	Quadratic	4.16×10^{-3}	6.9	0.19	0.43	3.59×10^{-3}	6.1	0.89	<0.001

	Power	3.80×10^{-3}	0.4	0.25	0.12	5.60×10^{-3}	5.8	0.82	<0.001
	Exponential	3.67×10^{-3}	0	0.27	0.10	3.31×10^{-4}	0	0.89	<0.001
BANFF	Linear	0.435	1.2	0.08	0.26	0.240	0	0.49	0.002
	Logarithmic	0.426	0.88	0.10	0.21	0.296	44.1	0.38	0.009
	Quadratic	0.435	4.69	0.08	0.54	0.227	43.1	0.52	0.006
	Power	0.405	0	0.15	0.13	0.321	45.5	0.33	0.02
	Exponential	0.409	0.17	0.14	0.15	0.279	43.1	0.41	0.006